Legendre Polynomials Application (Griffiths example 3.6, 3.7).

$V(\theta)$ specified on surface of hollow sphere, find potential inside sphere.

$$\Phi(R,\theta) = \sum_l \left(A_l R^l + B_l R^{-(l+1)}\right) P_l(\cos\theta).$$

$$= \sum_l A_l R^l P_l[\cos\theta] \quad \text{for origin included.}$$

Use $\langle P_m | P_l \rangle = \frac{2}{2l+1}\delta_{ml}$ over $[-1,1]$

$$A_l R^l \frac{2}{2l+1} = \int_0^\pi \Phi(R,\theta) P_l[\cos\theta] \sin\theta \, d\theta$$

$$\Rightarrow A_l r^l = \frac{2l+1}{2}\left(\frac{r}{R}\right)^l \int_0^\pi \Phi(R,\theta) P_l[\cos\theta] \sin\theta \, d\theta.$$

$$\Rightarrow \boxed{\Phi(r,\theta) = \sum_{l=0}^\infty \frac{2l+1}{2}\left(\frac{r}{R}\right)^l \left[\int_0^\pi \Phi(R,\theta) P_l[\cos\theta] \sin\theta \, d\theta\right] P_l[\cos\theta] \quad \text{for } r<R,\ \theta\in[0,\pi]}$$

Potential outside? make substitution

$$A_l R^l \frac{2}{2l+1} = \langle \Phi(R,\theta) | P_l[\cos\theta] \rangle \rightarrow B_l R^{-(l+1)} \frac{2}{2l+1} = \langle \cdots | \cdots \rangle.$$

$$\boxed{\Phi(r,\theta) = \sum_{l=0}^\infty \frac{2l+1}{2}\left(\frac{R}{r}\right)^{l+1} \left[\int_0^\pi \Phi(R,\theta) P_l[\cos\theta] \sin\theta \, d\theta\right] P_l \cos\theta}$$